In this section, we analytically derive a fundamental diagram of an urban rail transit system. This is a limitation of the current model. Note that scale of passenger-crowding can be excessive. Therefore, a regime with excessively large congestion and passenger-density is usually to eliminate bunching—in other words, such control makes the operation steady.

Macroscopic & Dynamic Model Based on FD

- Considers an exit-flow model with the FD as the exit-flow function
- Calculates train out-flow \( d(t) \) and passenger out-flow \( d_p(t) \), based on the FD function \( f(t) \) and initial and boundary conditions \( a(t) \), \( a_d(t) \), and \( TT(t) \)
- Notable feature of model is high tractability

Validation of Macroscopic Model

- Result of the microscopic model
  - Colored curves represent trajectories of each train that travels in upward direction while stopping at every station

- Result of the macroscopic model
  - Graphs show comparison between microscopic and macroscopic models
    - Macroscopic model reproduced results of the microscopic one fairly precisely
    - Congestion and delay during the peak time period were captured well

Fundamental Diagram of Railway Operation

- Microscopic assumptions on railway operation
  - Passenger boarding time is modeled using a bottleneck model:
    \[ t_p = \frac{v_p}{\mu_p} + g_b \]
  - Cruising behavior of train is modeled using the simplified car-following model of Newell (2002):
    \[ x_m(t) = \min\{x_m(t - \tau) + v_f t, x_m(t - \tau) - \delta\} \]

- Steady state of railway operation based on above assumptions

- Fundamental diagram (FD) of rail transit operation relating train-flow \( q \), train-density \( k \), and passenger-flow \( q_p \) under every steady state is expressed as:
  \[ Q(k, q_p) = \begin{cases} 
  \frac{lk - q_p/\mu_p}{g_b + 1/v_f} & \text{if } k < k^*(q_p) \\
  \frac{\delta}{(l - \delta)g_b + \tau} \left( k - k^*(q_p) \right) + q^*(q_p) & \text{if } k \geq k^*(q_p) 
  \end{cases} \]

- Numerical example of the FD
  - Piecewise linear relation (i.e. triangular)
  - Left side \( \rightarrow \) free-flowing regime
  - Top vertex \( \rightarrow \) critical regime
  - Right side \( \rightarrow \) congested regime

Macroscopic and Dynamic Model of Urban Rail Transit: Fundamental Diagram Approach

Background and Objective

Urban mass transit such as metro plays a significant role in transportation in metropolitan areas. Its most notable usage is the morning commute situation, in which excessive passenger demand is generated during a short time period.