Network Throughput Under Dynamic User Equilibrium: Queue Spillback, Paradox and Traffic Control

**Introduction**

**Macroscopic Fundamental Diagram (MFD):** A steady state functional relationship between vehicle accumulation and network throughput within certain networks.

MFD is useful for a robust network-wide traffic control but the microscopic mechanisms behind the macroscopic behaviors of an MFD are not completely understood.

**Purpose:** Clarify the relationship between an MFD and congestion patterns in a general network with one-to-many OD demands in dynamic user equilibrium.

**Framework**

An analytical formula for the network throughput is derived by solving a novel inverse problem of the DUE problem with a periodic boundary condition

- Through congestion patterns, the effects of network configurations and route choice behaviors can be incorporated into the analysis.

A sensitivity analysis of the formula reveals that

- Types of queue spillbacks that cause a decreasing in network throughput.
- Types of links that cause a capacity increasing paradox

Based on the revealed mechanisms, we explore a network control at merge for improving network throughput.

**Analytical formula**

Network throughputs are characterized by

- Reduced networks: topology, capacity, destination distributions
- Growth rate of OD travel times: \( \bar{\tau}_d \)

Congestion pattern

\[
\bar{\tau}_d = \sum_{i} \sum_{d} V_{id} \bar{\tau}_{id} - \sum_{i} \left( V_{id}(V_{id} - 1)(V_{id} \bar{\tau}_{id} - \delta_i) + \delta_i \right)
\]

\( V_{id} \) : Node-link incident matrix (reduced network) \( \bar{\tau}_{id} \) : capacity of link \( i \), \( d \) (subscript) : corresponding to transient and destination nodes

**Numerical examples**

Network comprising of an arterial road and bypasses

- Analytical formula agrees well with the simulation!!
- Revealed mechanisms actually happened during P1-4.

**Sensitivity analysis**

**Capacity decreasing mechanisms:** Queue spillback

1. Prevents flows from entering in destinations (i.e., decreasing of inflows to destinations)
2. Alters route choice pattern (i.e., increasing of flow passing through destinations)

- Capacity increasing paradox occurs when the increase of a link capacity increases of flow passing through destinations

**Proposed merge control** locally adjusts capacities of links at merge to reduce flow passing through destinations (i.e., increase the global network throughput)

- Capacity maximization policy
  - P0 (Smith, 1979) and proposed control achieve higher network throughput than no control case.
- Equi-saturation policy leads to an oscillation of allocation of capacities/route choice patterns.

**Summary and future works**

- Interaction of users with different destination are the main causes of decrease in the network throughput
- Proposed control may consistently improve network throughput
- Future works: Systematic numerical experiments, M2M ODs

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