Network Throughput Under Dynamic User Equilibrium: Queue Spillback, Paradox and Traffic Control

動的利用者均衡下におけるネットワークスループットの解析

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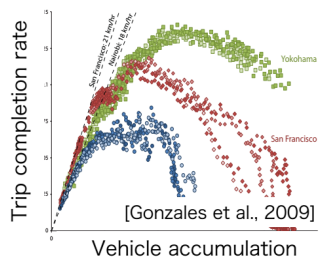
Introduction

Macroscopic Fundamental Diagram (MFD) : a steady state functional relationship between vehicle accumulation and network throughput within certain networks.

MFD is useful for a robust network-wide traffic control ***but*** the microscopic mechanisms behind the macroscopic behaviors of an MFD are not completely understood.

Purpose: Clarify the relationship between an MFD and congestion patterns in a general network with one-to-many OD demands in dynamic user equilibrium.



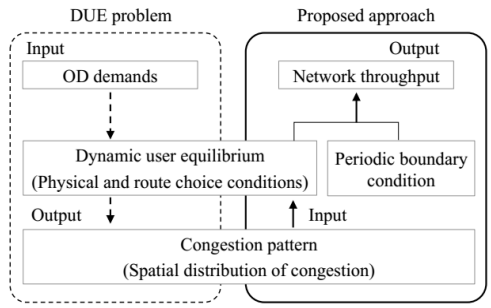
Framework

An **analytical formula for the network throughput** is derived by solving a novel inverse problem of the DUE problem with a periodic boundary condition

- Through congestion patterns, the **effects of network configurations** and **route choice behaviors** can be incorporated into the analysis.

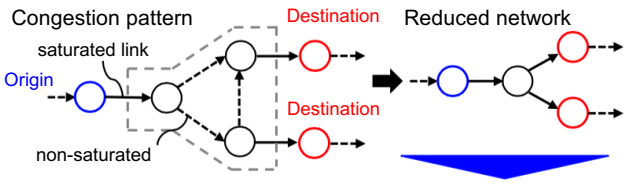
A **sensitivity analysis of the formula** reveals that
- Types of **queue spillbacks** that cause a decreasing in network throughput.
- Types of links that cause a **capacity increasing paradox**

Based on the revealed mechanisms, we explore **a network control at merge** for improving network throughput.



Analytical formula

Network throughputs are characterized by
- **Reduced networks**: topology, capacity, destination distributions
- Growth rate of **OD travel times:** $\bar{\tau}_d$



【Analytical Formula of Network Throughput (one-to-many)】

$$\bar{\mathbf{f}}_d = \mathbf{T}^{-1}\mathbf{V}_{dd}\bar{\boldsymbol{\tau}}_d - \mathbf{T}^{-1}[\mathbf{V}_{di}(\mathbf{V}_{ii})^{-1}(\mathbf{V}_{id}\bar{\boldsymbol{\tau}}_d - \boldsymbol{\delta}_i) + \boldsymbol{\delta}_d]$$

$\mathbf{V}_{ab} \equiv \mathbf{A}_a\mathbf{M}\mathbf{A}_b^{\mathrm{T}}, \mathbf{T} \equiv \mathrm{diag}[..., \bar{\tau}_d, ...], \mathbf{M} \equiv \mathrm{diag}[..., \mu_l, ...]$

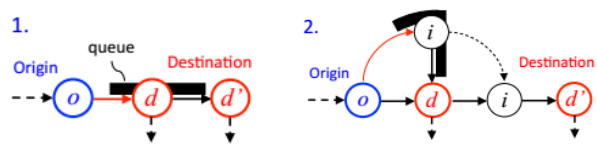
$\mathbf{A}_a$: Node-link incident matrix (reduced network) μ_l : capacity of link l

i, d (subscript) : corresponding to transient and destination nodes

Sensitivity analysis

Capacity decreasing mechanisms: *Queue spillback*

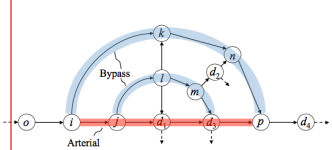
1. prevents flows from entering in destinations
 (i.e., **decreasing of inflows to destinations**)
2. alters route choice pattern
(i.e., **increasing of flow passing through destinations**)



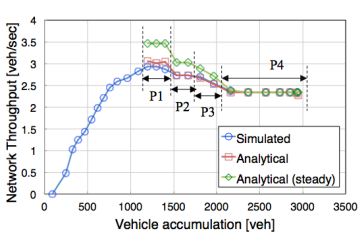
Capacity increasing paradox occurs when the increase of a link capacity increases of **flow passing through destinations**

Proposed merge control *locally* adjusts capacities of links at merge to reduce **flow passing through destinations** (i.e., increase the ***global*** network throughput)

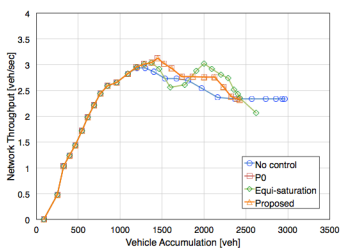
Numerical examples



Network comprising of an arterial road and bypasses



- Analytical formula agrees well with the simulation!!
- Revealed mechanisms actually happened during P1-4.



- **Capacity maximization policy P0** (Smith, 1979) and **proposed control** achieve higher network throughput than no control case.
- **Equi-saturation policy** leads to an **oscillation** of allocation of capacities/route choice patterns.

Summary and future works

- **Interaction of users with different destination** are the main causes of decrease in the network throughput
- Proposed control may consistently improve network throughput
- Future works: Systematic numerical experiments, M2M ODs