Continuum Car-following Model for Capacity Drop at Sag and Tunnel Bottlenecks

高速道路サグのボトルネック現象を表現する連続体追従モデル

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0. Introduction

In intercity expressways in Japan, nearly 80% of traffic congestion occurs at sags or uphill (60%) and tunnel entrances (20%). However, no appropriate model was established to describe congestion phenomena (see "Macroscopic Phenomena" part) at the sag and tunnel bottlenecks. To fill this research gap, we proposed a simple continuum car-following (CF) model of the **capacity drop** at such bottlenecks. By analyzing the proposed model, we uncovered the mechanism and properties of the evolution of the capacity drop.

1. Macroscopic phenomena & their behavioral mechanisms (conjectures)

found and conjectured by Koshi et al. (1992)

Capacity Reduction: Road capacity can be reduced compared with a normal section substantially.

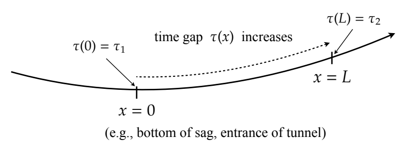
Behavioral mechanism: **A larger time distance (or time gap)** is needed comparing with normal section, on average. However, *the phenomenon is uncertain in general, its detailed mechanism is still unknown*.

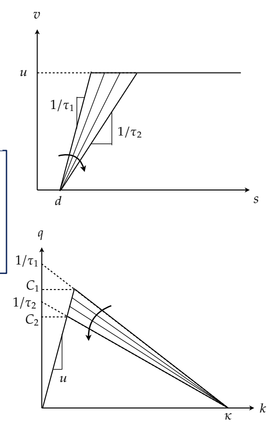
Capacity Drop: Discharge flow rate is further reduced once upstream queue forms (around 10%).

Behavioral mechanism: **An extremely low acceleration rate** at the downstream of the bottleneck (for 1-3 kilometers). *Nearly stationary and stable phenomenon.*

Modeling strategy: The capacity drop, which is relatively stable, should be described first.
cf. Existing models mainly focused on capacity reduction.

2. Continuum car-following (CF) model





Assumption:
Time gap increases inside bottleneck $x \in [0, L]$ (above Fig).
→ Left Figs: The resulting FDs

Spacing for continuum vehicles

$$s(t,n) \equiv \{X(t,n-\Delta n) - X(t,n)\}/\Delta n$$

Location dependent speed-spacing FD

$$V(s,x) = \min\left\{u, \left(s(t,n) - s_j\right)/\tau(X(t,n))\right\}$$

Proposed car-following model *… extension of KW model (Jin, 2018)*

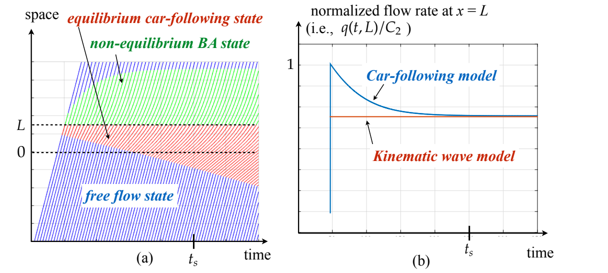
$$X(t+\Delta t, n) = X(t,n) + \min\left\{V(s,x), v(t,n) + A(v,x)\Delta t\right\}\Delta t$$

Bounded acceleration (BA) model (may lead to the capacity drop)

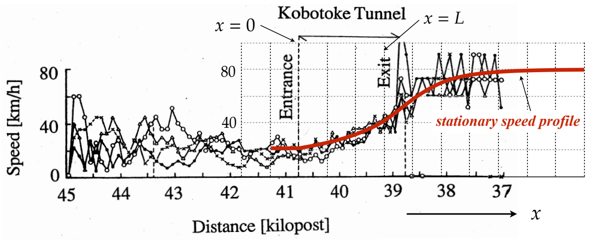
Location dependent FD (exogeneous) leads to the capacity reduction

3. Numerical example & validation

Simple lead-vehicle problem (leader: free-flow, followers: model)



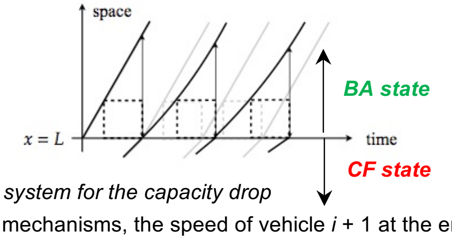
・The proposed CF model can describe the evolution of the capacity drop, while the KW model cannot (Fig. (b)).
・The evolution is solely related to the formation of the BA state at the downstream of the bottleneck (Fig. (a)).
・The stationary speed profile of the proposed CF model agrees well with the empirical data (Fig. below).



4. Theory of the capacity drop

Mechanism of the capacity drop (Steps 1 and 2 are repeated)

1. After passing the bottleneck, the followers cannot recover their speeds to the free-flow speed instantaneously due to the **BA model**.
2. The arriving time and the **equilibrium (or FD) speed** at the end of the bottleneck are delayed and reduced, which reduces the flow rate (or increases the headway).



Iterated function system for the capacity drop

… By the above mechanisms, the speed of vehicle *i* + 1 at the end of the bottleneck is predicted from that of vehicle *i*.

$$v_{i+1} = f(v_i) \equiv F(G(v_i)) = \left(\alpha\Delta n + 1/\sqrt{\beta\Delta n + v_i^2}\right)^{-1}$$

$$\text{where} \quad v_1 = F(u), \quad \alpha \equiv \Delta\tau/L, \quad \beta \equiv 2A^C(L)d.$$

Theorem. *The iterated function system has a unique capacity drop stationary state, and the state is stable in the sense that the system converges to it from any speed below the free-flow speed.*