# Modeling the Trajectories and Trajectory Variation of Left－Turning Vehicles at Signalized Intersections 

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## Introduction

Information regarding trajectories of turning vehicles at signalized intersections can be used in many applications such as in microscopic simulation tools developed for safety evaluations，motion planning of autonomous vehicles，and visualization of realistic vehicle movements in driving simulator applications．However，a proper framework to realistically model and estimate trajectories of turning vehicles is currently unavailable．This study explores the applicability of the minimum jerk concept，which has been applied in neuroscience and robotics domains，to model free－flow trajectories of turning vehicles．

## Modelling approach

## （1）Minimum－jerk concept by Flash and Hogan（1985）：

When moving a hand to an initial position to a final position within a given time duration $t_{f}$ ，and the cost to be minimized in order to maximize the smoothness of the trajectory is：

$$
J=\frac{1}{2} \int_{0}^{t_{f}}\left(\left(\frac{d^{3} x}{d t^{3}}\right)^{2}+\left(\frac{d^{3} y}{d t^{3}}\right)^{2}\right) d t
$$

Solution（Flash and Hogan 1985）：

$$
\begin{gathered}
x(t)=a_{0}+a_{1} t+a_{2} t^{2}+a_{3} t^{3}+a_{4} t^{4}+a_{5} t^{5} \\
y(t)=b_{0}+b_{1} t+b_{2} t^{2}+b_{3} t^{3}+b_{4} t^{4}+b_{5} t^{5}
\end{gathered}
$$

Where；$a_{j}$ and $b_{j}(j=\{0, \ldots, 5\}$ are constants
This system of equations can be solved with 12 boundary conditions．
$\rightarrow$ Location，velocity and acceleration vectors at the initial and final positions can provide 12 boundary conditions． However，$t_{f}$ is an unknown．

## Model verification

A Monte Carlo simulation with 100 different random seeds was conducted by randomizing entry and exit speeds and accelerations based on empirical data．

»Estimates and observations are well matched
Fig．1．Comparison of estimated and empirical movement times and trajectories （Suemori－Dori intersection）
$\rightarrow$ Incorporating Minimum speed and location of minimum speed models by Wolfermann et al．（2011）in minimum－jerk solution，an estimate for $t_{f}$ can be obtained．
（2）Minimum speed（ $v_{\text {min }}$ ）and location of minimum speed $\left(s_{\text {min }}\right)$ models by Wolfermann et al．（2011）：

| Normal Distribution | Parameters | $v_{\text {min }}$ | $s_{\text {min }}$ |
| :---: | :---: | :---: | :---: |
|  |  | $\mathrm{N}(\mu, \sigma)$ | $\mathrm{N}(\mu, \sigma)$ |
| $\mu$ | Constant | －0．301 | 1.42 |
|  | Entering speed（m／s） | 0.0908 | － |
|  | Corner radius（m） | 0.0607 | 0.586 |
|  | Intersection angle（deg） | 0.0387 | 0.0896 |
|  | Lateral exit distance（m） | 0.233 | 0.577 |
|  | Heavy vehicle dummy （HV：1，PC：0） | －0．496 | － |
| $\sigma$ | Constant | 0.665 | 0.135 |
|  | Corner radius（m） | － | 0.144 |
|  | Lateral exit distance（m） | 0.0419 | 0.336 |

## Conclusions

$\checkmark$ The proposed approach，which is based on minimum－jerk concept，can reproduce free－flow turning trajectories with a good accuracy．
$\checkmark$ Effects of geometric features（curve radii，intersection angle）on trajectories are also realistically captured．

