A Macroscopic and Dynamic Model of Urban Rail Transit: Fundamental Diagram Approach

都市鉄道の巨視的運行モデル：Fundamental Diagramアプローチ

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Background and Objective

Urban mass transit such as metro plays a significant role in transportation in metropolitan areas. Its most notable usage is the morning commute situation, in which excessive passenger demand is generated during a short time period.

Two types of congestion in rail transit

1) Train-congestion
Congestion involving consecutive trains using same tracks

2) Passenger-congestion
Congestion of passengers at station platforms

Develop analytical model of the dynamics of an urban rail transit

- Consider both types of congestion and physical interaction bet. them
- High analytical tractability
- Capable to obtain policy implications on management strategies

Image source: http://www.tourism-review.com/worlds-10-most-crowded-subways-news3987 (Accessed on 2017/05/14)

Fundamental Diagram of Railway Operation

- **Microscopic assumptions on railway operation**
  - Passenger boarding time is modeled using a bottleneck model:

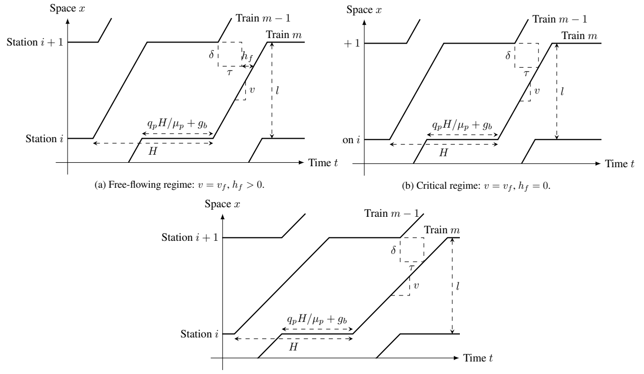
$$t_b = \frac{n_p}{\mu_p} + g_b$$

  - Cruising behavior of train is modeled using the simplified car-following model of Newell (2002):

$$x_m(t) = \min\{x_m(t-\tau) + v_f\tau, x_{m-1}(t-\tau) - \delta\}$$

Variables:
μ_p : passenger boarding flow-rate
g_b : buffer time (time for door opening/closing)
n_p : no. of waiting passengers at station
$x_m(t)$: position of a train m at time t
$m-1$: indicates preceeding train of train m
τ : physical minimum headway time
v_f :free-flow speed
δ : minimum spacing
h_f : buffer headway time
H : headway time bet. each successive trains
l : distance bet. each adjacent stations
v : cruising speed of all the trains
q_p : passenger-flow to each station

- **Steady state of railway operation based on above assumptions**



(a) Free-flowing regime: $v = v_f, h_f > 0$.

(b) Critical regime: $v = v_f, h_f = 0$.

(c) Congested regime: $v < v_f, h_f = 0$.

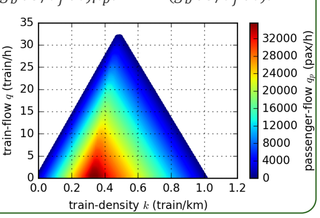
- **Fundamental diagram (FD)** of rail transit operation relating train-flow q, train-density k, and passenger-flow q_p under every steady state is expressed as:

$$Q(k, q_p) = \begin{cases} \dfrac{lk - q_p/\mu_p}{g_b + l/v_f}, & \text{if } k < k^*(q_p) \\ -\dfrac{l\delta}{(l-\delta)g_b + \tau l}\left(k - k^*(q_p)\right) + q^*(q_p), & \text{if } k \geq k^*(q_p) \end{cases}$$

$q^*(q_p)$ and $k^*(q_p)$ are train-flow and train-density, respectively, in a critical state with q_p

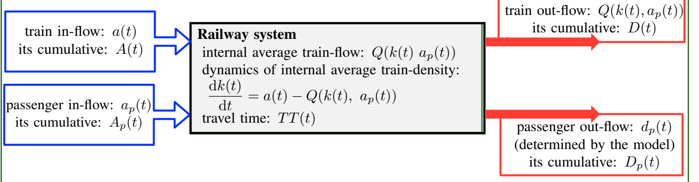
$$q^*(q_p) = \frac{1 - q_p/\mu_p}{g_b + \delta/v_f + \tau'} \qquad k^*(q_p) = \frac{(l-\delta)/v_f - \tau}{(g_b + \delta/v_f + \tau)\mu_p l} q_p + \frac{g_b + l/v_f}{(g_b + \delta/v_f + \tau)l}$$

- **Numerical example of the FD**
  - Piecewise linear relation (i.e. triangular)
  - Left side ➔ free-flowing regime
  - Top vertex ➔ critical regime
  - Right side ➔ congested regime



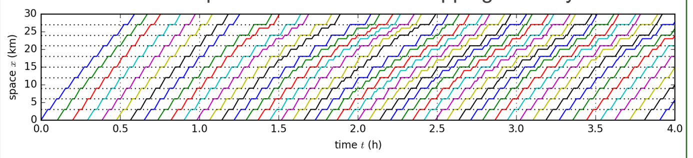
Macroscopic & Dynamic Model Based on FD

- Considers an **exit-flow model** with the FD as the exit-flow function
- Calculates train **out-flow** $d(t)$ and passenger **out-flow** $d_p(t)$, based on the **FD function** $Q(\cdot)$ and initial and boundary conditions $a(t)$, $a_p(t)$, and $TT(0)$
- Notable feature of model is **high tractability**

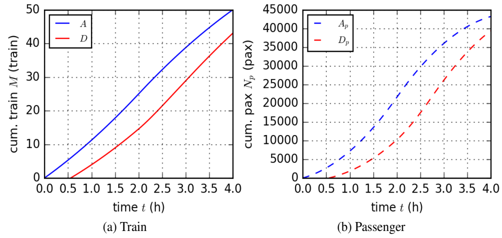


Validation of Macroscopic Model

- **Result of the microscopic model**
  - Colored curves represent trajectories of each train that travels in upward direction while stopping at every station

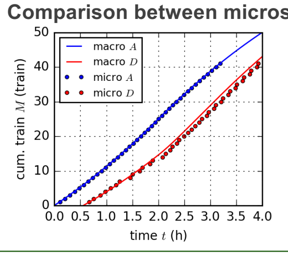


- **Result of the macroscopic model**



(a) Train (b) Passenger

- **Comparison between microscopic and macroscopic models**



- Macroscopic model reproduced results of the microscopic one fairly precisely
- Congestion and delay during the peak time period were captured well